ONLINE WORKSHEET PACKAGE

ALGEBRA 2

POLYNOMIAL FUNCTION

- Factor Theorem
- Rational Root Theorem
- Remainder Theorem

DR AHN MATH & LEARNING CENTER

YouTube

$$f(x) = (x+3)x(x-2)(x-4)$$

- $(x+3),\ x,\ (x-2),\ (x-4) \Rightarrow factors$
- $f(-3)=0,\ f(0)=0,\ f(2)=0,\ f(4)=0$
- $-3,\ 0,\ 2,\ 4 \Rightarrow Zeros$
- $-3,\ 0,\ 2,\ 4 \Rightarrow x\ intercepts$

- Factor Theorem

$$f(c)=0 \Rightarrow (x-c)\ is\ a\ factor$$
$$(x-c)\ is\ a\ factor \Rightarrow f(c)=0$$

$$f(2)=0 \Rightarrow (x-2)\ is\ a\ factor$$
$$f(-3)=0 \Rightarrow (x+3)\ is\ a\ factor$$
$$f(0)=0 \Rightarrow x\ is\ a\ factor$$

$$(x+1)\ is\ a\ factor \Rightarrow f(-1)=0$$
$$(x-5)\ is\ a\ factor \Rightarrow f(5)=0$$
$$x\ is\ a\ factor \Rightarrow f(0)=0$$

YouTube

Factoring Polynomial

- $f(x) = x^3 + 3x^2 - 16x - 48, \quad -3$, one zero given

- $f(x) = x^3 - 6x^2 - 15x + 100, \quad 5$, one zero given

Factoring Polynomial

- $f(x) = x^3 + 3x^2 - 16x - 48, \quad -3$, one zero given

$$= x^2(x+3) - 16(x+3)$$
$$= (x^2 - 16)(x+3)$$
$$= (x+4)(x-4)(x+3)$$

$f(-3) = 0 \quad \rightarrow \quad (x+3) \quad is \quad a \quad factor$
$f(x) = (x+3)(\qquad\qquad)$

-3	1	3	-16	-48
		-3	0	48
	1	0	-16	0

$$f(x) = (x+3)(x^2 - 16)$$
$$= (x+3)(x+4)(x-4)$$

- $f(x) = x^3 - 6x^2 - 15x + 100, \quad 5$, one zero given

$f(5) = 0 \quad \rightarrow \quad (x-5) \quad is \quad a \quad factor$
$f(x) = (x-5)(\qquad\qquad)$

5	1	-6	-15	100
		5	-5	-100
	1	-1	-20	0

$$f(x) = (x-5)(x^2 - x - 20)$$
$$= (x-5)(x-5)(x+4)$$
$$= (x-5)^2(x+4)$$

MODULE 6 Name________________________________

FACTOR THEOREM & SYNTHETIC DIVISION

Factor each. One zero has been given.

1) $f(x) = x^3 - 6x^2 - 15x + 100;\ 5$

2) $f(x) = x^3 + x^2 - 22x - 40;\ 5$

3) $f(x) = x^3 - x^2 - 8x + 12;\ -3$

4) $f(x) = x^3 - 8x^2 + 19x - 12;\ 3$

5) $f(x) = x^4 - 7x^3 + 16x^2 - 12x;\ 3$

6) $f(x) = x^3 + 3x^2 - 16x - 48;\ -3$

7) $f(x) = x^4 + 3x^3 - 18x^2 - 40x;\ -5$

8) $f(x) = x^3 - 8x^2 + 20x - 16;\ 2$

9) $f(x) = x^3 - 4x^2 - 25x + 100;\ -5$

10) $f(x) = x^3 + 3x^2 - 4x - 12;\ 2$

11) $f(x) = x^4 - x^3 - 10x^2 - 8x;\ -2$

12) $f(x) = x^3 + x^2 - 4x - 4;\ -2$

13) $f(x) = x^3 + 8x^2 + 20x + 16;\ -2$

14) $f(x) = x^3 - 10x^2 + 33x - 36;\ 3$

Answers to FACTOR THEOREM & SYNTHETIC DIVISION

1) $f(x) = (x-5)^2(x+4)$

2) $f(x) = (x+4)(x+2)(x-5)$

3) $f(x) = (x-2)^2(x+3)$

4) $f(x) = (x-1)(x-4)(x-3)$

5) $f(x) = x(x-2)^2(x-3)$

6) $f(x) = (x-4)(x+4)(x+3)$

7) $f(x) = x(x-4)(x+2)(x+5)$

8) $f(x) = (x-2)^2(x-4)$

9) $f(x) = (x-5)(x-4)(x+5)$

10) $f(x) = (x+2)(x+3)(x-2)$

11) $f(x) = x(x-4)(x+1)(x+2)$

12) $f(x) = (x+1)(x-2)(x+2)$

13) $f(x) = (x+2)^2(x+4)$

14) $f(x) = (x-4)(x-3)^2$

YouTube

Factoring Polynomial

- One zero is not given

  $\rightarrow$ Find one zero by ‘Guess and Check’

  $\rightarrow$ Possible zeros (Rational Root(Zero) Theorem)

Rational Root Theorem

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

$$\Rightarrow \textit{Possible zeros} = \pm\frac{P}{q} \quad \left\langle \begin{array}{l} \text{P : factors of } a_0 \\ \\ \text{q : factor of } a_n \end{array} \right.$$

$$f(x) = 3x^3 - 5x^2 + 1000x - 4$$

$$\Rightarrow \textit{Possible zeros} \qquad \pm\frac{1,\ 2,\ 3}{1,\ 3}$$

$$= \pm 1,\ \pm\frac{1}{3},\ \ \pm 2,\ \ \pm\frac{2}{3},\ \ \pm 4,\ \ \pm\frac{4}{3}$$

YouTube

Factoring Polynomial

$$f(x) = x^4 - 6x^3 - 20x^2 - 42x - 189$$

- Rational Roots Theorem $\pm\dfrac{1,\ 3,\ 7,\ 9,\ 21,\ 27,\ 63,\ 189}{\pm 1}$

$$= \pm 1,\ \pm 3,\ \pm 7,\ \pm 9,\ \pm 21,\ \pm 27,\ \pm 63,\ \pm 189$$

- Factor Theorem / Synthetic Division

$$\begin{array}{r|rrrrr} -3 & 1 & -6 & -20 & -42 & -189 \\ & 0 & -3 & 27 & -21 & 189 \\ \hline & 1 & -9 & 7 & -63 & 0 \end{array} \quad \Rightarrow \quad (x+3)\ is\ a\ factor$$

$$\begin{array}{r|rrrr} 9 & 1 & -9 & 7 & -63 \\ & 0 & 9 & 0 & 63 \\ \hline & 1 & 0 & 7 & 0 \end{array} \quad \Rightarrow \quad (x-9)\ is\ a\ factor$$

$$f(x) = (x+3)(x-9)(x^2+7)$$

- Quadratic factoring

$$f(x) = (x+3)(x-9)(x+\sqrt{7}i)(x-\sqrt{7}i)$$

$$Zeros:\ -3,\ 9,\ \pm\sqrt{7}i$$

YouTube

- $f(x) = x^3 + 7x^2 + 2x - 40$

- $f(x) = x^4 - x^3 - 16x^2 + 4x + 48$

- $f(x) = x^3 + 7x^2 + 2x - 40$
  - *Possible Zeros* $= \pm 1, \pm 2, \pm 4, \pm 5, \pm 8, \pm 10, \pm 20, \pm 40$

$$f(1) = 1 + 7 + 2 - 40 \neq 0$$
$$f(-1) = -1 + 7 - 2 - 40 \neq 0$$
$$f(2) = 8 + 28 + 4 - 40 = 0$$

$$\begin{array}{r|rrrr} 2 & 1 & 7 & 2 & -40 \\ & & 2 & 18 & 40 \\ \hline & 1 & 9 & 20 & \boxed{0} \end{array}$$

$$\begin{aligned} f(x) &= (x-2)(x^2+9x+20) \\ &= (x-2)(x+4)(x+5) \end{aligned}$$

- $f(x) = x^4 - x^3 - 16x^2 + 4x + 48$
  - *Possible Zeros* $= \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48$

$$\begin{array}{r|rrrrr} 1 & 1 & -1 & -16 & 4 & 48 \\ & & 1 & 0 & -16 & -12 \\ \hline & 1 & 0 & -16 & -12 & 36 \end{array}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -1 & -16 & 4 & 48 \\ & & 2 & 2 & -28 & -48 \\ \hline & 1 & 1 & -14 & -24 & \boxed{0} \end{array}$$

$$f(x) = (x-2)(x^3+x^2-14x-24)$$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -14 & -24 \\ & & -1 & 0 & 14 \\ \hline & 1 & 0 & -14 & -10 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -14 & -24 \\ & & -2 & 2 & 24 \\ \hline & 1 & -1 & -12 & \boxed{0} \end{array}$$

$$\begin{aligned} f(x) &= (x-2)(x+2)(x^2-x-12) \\ &= (x-2)(x+2)(x-4)(x+3) \end{aligned}$$

Name____________________________________

The Rational Root Theorem

Date________________ Period____

State the possible rational zeros for each function.

1) $f(x) = 3x^2 + 2x - 1$

2) $f(x) = x^6 - 64$

3) $f(x) = x^2 + 8x + 10$

4) $f(x) = 5x^3 - 2x^2 + 20x - 8$

5) $f(x) = 4x^5 - 2x^4 + 30x^3 - 15x^2 + 50x - 25$

6) $f(x) = 5x^4 + 32x^2 - 21$

7) $f(x) = x^3 - 27$

8) $f(x) = 2x^4 - 9x^2 + 7$

Name________________________________

The Rational Root Theorem

Date________________ Period____

State the possible rational zeros for each function.

1) $f(x) = 3x^2 + 2x - 1$

$\pm 1, \pm \frac{1}{3}$

2) $f(x) = x^6 - 64$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$

3) $f(x) = x^2 + 8x + 10$

$\pm 1, \pm 2, \pm 5, \pm 10$

4) $f(x) = 5x^3 - 2x^2 + 20x - 8$

$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{5}, \pm \frac{2}{5}, \pm \frac{4}{5}, \pm \frac{8}{5}$

5) $f(x) = 4x^5 - 2x^4 + 30x^3 - 15x^2 + 50x - 25$

$\pm 1, \pm 5, \pm 25, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{25}{2}, \pm \frac{1}{4}, \pm \frac{5}{4}, \pm \frac{25}{4}$

6) $f(x) = 5x^4 + 32x^2 - 21$

$\pm 1, \pm 3, \pm 7, \pm 21, \pm \frac{1}{5}, \pm \frac{3}{5}, \pm \frac{7}{5}, \pm \frac{21}{5}$

7) $f(x) = x^3 - 27$

$\pm 1, \pm 3, \pm 9, \pm 27$

8) $f(x) = 2x^4 - 9x^2 + 7$

$\pm 1, \pm 7, \pm \frac{1}{2}, \pm \frac{7}{2}$

15) $f(x) = x^3 + 7x^2 + 2x - 40$

16) $f(x) = x^4 - 2x^3 - 5x^2 + 6x$

17) $f(x) = x^3 - 10x^2 + 31x - 30$

18) $f(x) = x^3 + 2x^2 - 23x - 60$

19) $f(x) = x^3 - 2x^2 - 9x + 18$

20) $f(x) = x^3 - 7x^2 + 14x - 8$

21) $f(x) = x^3 - 7x^2 + 15x - 9$

22) $f(x) = x^3 + 8x^2 + 19x + 12$

23) $f(x) = x^3 - 7x^2 + 7x + 15$

24) $f(x) = x^3 - 12x^2 + 47x - 60$

25) $f(x) = x^3 - 13x^2 + 55x - 75$

26) $f(x) = x^4 + 5x^3 - 9x^2 - 45x$

27) $f(x) = x^3 - 4x^2 - 7x + 10$

28) $f(x) = x^3 + 6x^2 + 11x + 6$

29) $f(x) = x^4 + 4x^3 - 4x^2 - 16x$

30) $f(x) = x^3 - 11x^2 + 40x - 48$

Answers to FACTOR THEOREM & SYNTHETIC DIVISION

13) $f(x) = (x+2)^2(x+4)$

14) $f(x) = (x-4)(x-3)^2$

15) $f(x) = (x+5)(x+4)(x-2)$

16) $f(x) = x(x-3)(x-1)(x+2)$

17) $f(x) = (x-3)(x-2)(x-5)$

18) $f(x) = (x-5)(x+4)(x+3)$

19) $f(x) = (x+3)(x-2)(x-3)$

20) $f(x) = (x-1)(x-4)(x-2)$

21) $f(x) = (x-3)^2(x-1)$

22) $f(x) = (x+1)(x+4)(x+3)$

23) $f(x) = (x+1)(x-5)(x-3)$

24) $f(x) = (x-4)(x-5)(x-3)$

25) $f(x) = (x-5)^2(x-3)$

26) $f(x) = x(x-3)(x+5)(x+3)$

27) $f(x) = (x-1)(x+2)(x-5)$

28) $f(x) = (x+1)(x+3)(x+2)$

29) $f(x) = x(x-2)(x+4)(x+2)$

30) $f(x) = (x-4)^2(x-3)$

YouTube

Remainder Theorem

$f(x)$ is divided by $(x-c) \rightarrow$ Remainder $\;r = f(c)$

- $f(x) = x^3 + 4x^2 - 5x + 3$ is divided by $(x+2) \rightarrow$ *Remainder* r?

-2	1	4	-5	3
		-2	-4	18
	1	2	-9	21

$$f(-2) = -8 + 16 + 10 + 3 = 21$$

$f(x)$ is divided by $(x-3) \rightarrow$ Remainder $r = f(3)$

$f(x)$ is divided by $(x+1) \rightarrow$ Remainder $r = f(-1)$

$f(x)$ is divided by $x \rightarrow$ Remainder $r = f(0)$

$f(3) = 2 \rightarrow$ when $f(x)$ is divided by $(x-3)$, Remainder $r = 2$

$f(0) = 4 \rightarrow$ when $f(x)$ is divided by x, Remainder $r = 4$

$f(-2) = -1 \rightarrow$ when $f(x)$ is divided by $(x+2)$, Remainder $r = -1$

Remainder Theorem Practices :

1. Without using long division, find each remainder :

(a) $(2x^2+6x+8)\div(x+1)$

(b) $(x^2+4x+12)\div(x-4)$

(c) $(x^3+6x^2-4x+3)\div(x+2)$

(d) $(3x^3+7x^2-2x-11)\div(x-2)$

2. Find each remainder :

(a) $(2x^2+x-6)\div(x+2)$

(b) $(x^3+6x^2-4x+2)\div(x+1)$

(c) $(x^3+x^2-12x-13)\div(x-2)$

(d) $(x^4-x^3-3x^2+4x+2)\div(x+2)$

3. When x^3+kx^2-4x+2 is divied by $x+2$ the remainder is 26, find k.

4. When $2x^3-3x^2+kx-1$ is divied by $x-1$ the remainder is 2, find k.

Remainder Theorem Practices :

1. Without using long division, find each remainder :

(a) $(2x^2+6x+8)\div(x+1)$

$f(-1)=2-6+8=4$

(b) $(x^2+4x+12)\div(x-4)$

$f(4)=16+16+12=44$

(c) $(x^3+6x^2-4x+3)\div(x+2)$

$f(-2)=-8+24+8+3=27$

(d) $(3x^3+7x^2-2x-11)\div(x-2)$

$f(2)=24+28-4-11=37$

2. Find each remainder :

(a) $(2x^2+x-6)\div(x+2)$

$f(-2)=8-2-6=0$

(b) $(x^3+6x^2-4x+2)\div(x+1)$

$f(-1)=-1+6+4+2=11$

(c) $(x^3+x^2-12x-13)\div(x-2)$

$f(2)=8+4-24-13=-25$

(d) $(x^4-x^3-3x^2+4x+2)\div(x+2)$

$f(-2)=16+8-12-8+2=6$

3. When x^3+kx^2-4x+2 is divied by $x+2$ the remainder is 26, find k.

$f(-2)=26$ $\qquad -8+4k+8+2=26$

$4k=24 \rightarrow k=6$

4. When $2x^3-3x^2+kx-1$ is divied by $x-1$ the remainder is 2, find k.

$f(1)=2$ $\qquad 2-3+k-1=2$

$k-2=2$

$k=4$

Accelerated Pre-Calculus

The Real Zeros of a Polynomial Function

I. **The Remainder Theorem**

If $f(x)$ is a polynomial function and if $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

II. **The Factor Theorem**

Let $f(x)$ be a polynomial function.

A. If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.

B. If $x - c$ is a factor of $f(x)$, then $f(c) = 0$.

1. Find the remainder when $p(x) = x^{15} + 3x^{10} + 2$ is divided by $x - 1$.

2. Find the quotient and remainder when the first polynomial is divided by the second.
   $x^3 - 2x - 18$; $x - 2$

3. Determine whether $(x - 2)$ or $(x + 2)$ is a factor of $x^5 - 3x^2 - 20$.

4. When a polynomial $p(x)$ is divided by $(x + 3)$, the quotient is $2x^2 - 3x + 9$ and the remainder is -11. Find $p(x)$.

Given a polynomial and one or more of its roots, find the remaining roots.

5. $2x^3 + 5x^2 - 23x + 10 = 0$; root $x = 2$

6. $2x^4 + 5x^3 - 8x^2 - 17x - 6$; roots $x = -1$ and $x = 2$

Accelerated Pre-Calculus

The Real Zeros of a Polynomial Function

I. The Remainder Theorem

If $f(x)$ is a polynomial function and if $f(x)$ is divided by $x-c$, then the remainder is $f(c)$.

II. The Factor Theorem

Let $f(x)$ be a polynomial function.

A. If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.

B. If $x - c$ is a factor of $f(x)$, then $f(c) = 0$.

1. Find the remainder when $p(x) = x^{15} + 3x^{10} + 2$ is divided by $x - 1$.

$P(1) = 1^{15} + 3(1)^{10} + 2 = 6$

2. Find the quotient and remainder when the first polynomial is divided by the second.
$x^3 - 2x - 18$; $x - 2$

2	1	0	-2	-18
		2	4	4
	1	2	2	-14

$x^2 + 2x + 2 - \frac{14}{x-2}$

3. Determine whether $(x - 2)$ or $(x + 2)$ is a factor of $x^5 - 3x^2 - 20$.

2	1	0	0	-3	0	-20
		2	4	8	10	20
	1	2	4	5	10	0

x-2 yes

-2	1	0	0	-3	0	-20
		-2	4	-8	22	-44
	1	-2	4	-11	22	-64

no

4. When a polynomial $p(x)$ is divided by $(x + 3)$, the quotient is $2x^2 - 3x + 9$ and the remainder is -11. Find $p(x)$.

$p(x) = (x+3)(2x^2 - 3x + 9) - 11$

$= 2x^3 - 3x^2 + 9x + 6x^2 - 9x + 27 - 11$

$p(x) = 2x^3 + 3x^2 + 16$

Given a polynomial and one or more of its roots, find the remaining roots.

5. $2x^3 + 5x^2 - 23x + 10 = 0$; root $x = 2$

2	2	5	-23	10
		4	18	-10
	2	9	-5	0

$2x^2 + 9x - 5$

$(2x - 1)(x + 5)$

$x = \frac{1}{2}, -5$

6. $2x^4 + 5x^3 - 8x^2 - 17x - 6$; roots $x = -1$ and $x = 2$

-1	2	5	-8	-17	-6
		-2	-3	11	6
2	2	3	-11	-6	0
		4	14	6	
	2	7	3	0	

$2x^2 + 7x + 3$

$(2x + 1)(x + 3)$

$x = -\frac{1}{2}, -3$

The Remainder Theorem

1) Find the remainder when $x^3 - 2x^2 - x - 2$ is divided by $x + 1$.

2) If $f(x) = x^3 + 3x - 4$. Find the remainder when $f(x)$ is divided by $x - 4$.

3) Find the remainder when $x^3 + 3x - 4$ is divided by $x + 1$.

4) Given that $f(x) = 6x^3 - 3x^2 - 17x + 7$, divide $f(x)$ by $x - 3$.

5) Find the remainder when $6x^3 + 27x^2 - 14x + 15$ is divided by $x + 5$.

6) When divided by $(x + 1)$ and $(x + 2)$, the expression

$$ax^2 + bx + 3$$

leaves remainders 6 and 9 respectively. Find the values for a and b

7) Find the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $x + 2$.

Contact: mphowehnzembayie@gmail.com

The Remainder Theorem

Answers

1) Find the remainder when $x^3 - 2x^2 - x - 2$ is divided by $x + 1$.

- 4

2) If $f(x) = x^3 + 3x - 4$. Find the remainder when $f(x)$ is divided by $x - 4$.

72

3) Find the remainder when $x^3 + 3x - 4$ is divided by $x + 1$.

- 8

4) Given that $f(x) = 6x^3 - 3x^2 - 17x + 7$, divide $f(x)$ by $x - 3$.

91

5) Find the remainder when $6x^3 + 27x^2 - 14x + 15$ is divided by $x + 5$.

10

6) When divided by $(x + 1)$ and $(x + 2)$, the expression

$$ax^2 + bx + 3$$

leaves remainders 6 and 9 respectively. Find the values for a and b

a=2, b= 1

7) Find the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $x + 2$.

8

Math 11UE - Mr. Duncan Name________________________________

4.3 Remainder Theorem - Practice Problems

Date________________

Find the remainder of the given division problem. Solve using Synthetic Division, and check by using Remainder Theorem.

1) $(v^4 - 7v^3 + 14v^2 - 13v + 19) \div (v - 4)$

2) $(x^3 + 10x^2 - 4) \div (x + 10)$

Find the remainder.

3) $(x^3 + 11x^2 + 24x + 9) \div (x + 2)$

4) $(x^4 + 14x^3 + 52x^2 + 27x - 39) \div (x + 8)$

Solve for *k*.

5) When $x^3 + kx + 1$ is divided by $x - 2$, the remainder is -3.

6) When $2x^4 + kx^2 - 3x + 5$ is divided by $x - 2$, the remainder is 3.

Solve.

7) When $x^3 + kx^2 - 2x - 7$ is divided by $x + 1$, the remainder is 5. What is the remainder when it is divided by $x - 1$?

8) When the polynomial $x^n + x - 8$ is divided by $x - 2$ the remainder is 10. What is the value of n ?

Answers to 4.3 Remainder Theorem - Practice Problems (ID: 1)

1) $v^3 - 3v^2 + 2v - 5$, R -1
2) x^2, R -4
3) $x^2 + 9x + 6$, R -3
4) $x^3 + 6x^2 + 4x - 5$, R 1
5) $k = -6$
6) $k = -7$
7) $k = 11$, $R = 3$
8) $n = 4$

WORKSHEET 2.1 & 2.2

1.10.1 Remainder Theorem and Factor Theorem

Remainder Theorem:
When a polynomial $f(x)$ is divided by $x-a$, the remainder is $f(a)$

1. Find the remainder when $2x^3+3x^2-17x-30$ is divided by each of the following:

   (a) $x-1$ (b) $x-2$ (c) $x-3$

   (d) $x+1$ (e) $x+2$ (f) $x+3$

Factor Theorem:
If $x=a$ is substituted into a polynomial for x, and the remainder is 0, then $x-a$ is a factor of the polynomial.

2. Using the above Theorem and your results from question 1 which of the given binomials are factors of $2x^3+3x^2-17x-30$?

3. Using the binomials you determined were factors of $2x^3+3x^2-17x-30$, complete the division (i.e. divide $2x^3+3x^2-17x-30$ by your chosen ($x-a$) and remember to fully factor your result in each case.

1.10.1 Remainder Theorem and Factor Theorem (Answers)

1. Find the remainder when $2x^3+3x^2-17x-30$ is divided by each of the following:

(a) $x-1$
$\therefore a=1$
$f(1)=2(1)^3+3(1)^2-17(1)-30$
$f(1)=2+3-17-30$
$f(1)=-42$

(b) $x-2$
$a=2$
$f(a)=-36$

(c) $x-3$
$a=3$
$f(a)=0$

(d) $x+1$
$a=-1$
$f(a)=-12$

(e) $x+2$
$a=-2$
$f(a)=0$

(f) $x+3$
$a=-3$
$f(a)=-6$

2. Using the above Theorem and your results from question 1 which of the given binomials are factors of $2x^3+3x^2-17x-30$?

From results → (c) $x-3$ and (e) $x+2$ are factors

3. Using the binomials you determined were factors of $2x^3+3x^2-17x-30$ complete the division (i.e. divide $2x^3+3x^2-17x-30$ by your chosen $x-a$) and remember to fully factor your result in each case.

(c) $x-3$

$$
\begin{array}{r}
2x^2+9x+10 \\
x-3\,\big)\,\overline{2x^3+3x^2-17x-30} \\
\underline{2x^3-6x^2} \qquad\downarrow\quad\downarrow \\
9x^2-17x\quad\downarrow \\
\underline{9x^2-27x}\quad\downarrow \\
10x-30 \\
\underline{10x-30} \\
0
\end{array}
$$

Result: $(x-3)(2x^2+9x+10)$
$(x-3)(2x+5)(x+2)$

(e) $x+2$

$$
\begin{array}{r}
2x^2-x-15 \\
x+2\,\big)\,\overline{2x^3+3x^2-17x-30} \\
\underline{2x^3+4x^2} \qquad\downarrow\quad\downarrow \\
-x^2-17x\quad\downarrow \\
\underline{-x^2-2x}\quad\downarrow \\
-15x-30 \\
\underline{-15x-30} \\
0
\end{array}
$$

Result: $(x+2)(2x^2-x-15)$
$(x+2)(2x+5)(x-3)$

(Note: The results are the same just rearranged.)

Tip 23 Remainder Theorem

When polynomial $f(x)$ is divided by $(x-a)$, the remainder R is equal to $f(a)$.
Polynomial $f(x)$ can be expressed as follows.

$$f(x)=(x-a)Q(x)+R, \text{ where } Q(x) \text{ is the quotient and } R \text{ is the remainder.}$$

The identical equation above is true for all values of x, especially $x=a$.
Therefore, $f(a)=(a-a)Q(a)+R \rightarrow f(a)=R$.

Examples:

1) Interpretation of $f(2)=5 \quad \rightarrow \quad$ The remainder is 5 when $f(x)$ is divided by $(x-2)$.
2) Interpretation of $f(-5)=-3 \quad \rightarrow \quad$ The remainder is -3 when $f(x)$ is divided by $(x+5)$.

SAT Practice

1. When $f(x)=x^2+3x+k$ is divided by $x-3$, the remainder is 25. What is the value of k?

---

2. What is the remainder when x^3-x^2-3x-1 is divided by $(x+3)$?

A) -36 B) -28 C) 14 D) 36

---

3. Find the value of k for which the remainder is zero when x^3-5x^2+x+k is divided by $(x-1)$?

TIPS

Tip 23 Remainder Theorem

When polynomial $f(x)$ is divided by $(x-a)$, the remainder R is equal to $f(a)$.
Polynomial $f(x)$ can be expressed as follows.

$$f(x)=(x-a)Q(x)+R, \text{ where } Q(x) \text{ is the quotient and } R \text{ is the remainder.}$$

The identical equation above is true for all values of x, especially $x=a$.
Therefore, $f(a)=(a-a)Q(a)+R \rightarrow f(a)=R$.

Examples:

1) Interpretation of $f(2)=5$ $\rightarrow$ The remainder is 5 when $f(x)$ is divided by $(x-2)$.
2) Interpretation of $f(-5)=-3$ $\rightarrow$ The remainder is -3 when $f(x)$ is divided by $(x+5)$.

SAT Practice

1. When $f(x)=x^2+3x+k$ is divided by $x-3$, the remainder is 25. What is the value of k?

$f(3)=25$ $\quad 9+9+k=25$

$k+18=25 \rightarrow k=7$

---

2. What is the remainder when x^3-x^2-3x-1 is divided by $(x+3)$?

A) -36 B) -28 C) 14 D) 36

$f(-3)=-27-9+9-1=-28$

---

3. Find the value of k for which the remainder is zero when x^3-5x^2+x+k is divided by $(x-1)$?

$f(1)=1-5+1+k=0$

$k-3=0$

$k=3$

Tip 24 Factor Theorem

If $f(a)=0$, then $f(x)$ has a factor of $(x-a)$.
$f(x)$ can be expressed with a factor of $(x-a)$ as follows.

$$f(x)=(x-a)Q(x)$$

Therefore, $f(a)=0$ means that the remainder is 0.

SAT Practice

1. If $(x-3)$ is a factor of x^3-4x+k, what is the value of the constant k?

A) -15 B) -10 C) 10 D) 15

---

2. If a polynomial $P(x)=x^2+kx-8$ has a factor of $(x-2)$, what is the value of k?

---

3. What is the value of k when $P(x)=x^3-5x^2-x+k$ is divisible by $x+1$?

A) -5
B) -2
C) 2
D) 5

---

4. What is the value of a when $g(x)=x^3+ax+b$ is divisible by $(x-1)(x-2)$?

A) -7
B) -4
C) 4
D) 8

TIPS

Tip 24 Factor Theorem

If $f(a)=0$, then $f(x)$ has a factor of $(x-a)$.
$f(x)$ can be expressed with a factor of $(x-a)$ as follows.

$$f(x)=(x-a)Q(x)$$

Therefore, $f(a)=0$ means that the remainder is 0.

SAT Practice

1. If $(x-3)$ is a factor of x^3-4x+k, what is the value of the constant k?

A) −15 B) −10 C) 10 D) 15

$f(3)=0$
$27-12+k=0$
$k+15=0 \rightarrow k=-15$

2. If a polynomial $P(x)=x^2+kx-8$ has a factor of $(x-2)$, what is the value of k?

2 $P(2)=0$ | $4+2k-8=0$ $2k=4 \rightarrow k=2$

3. What is the value of k when $P(x)=x^3-5x^2-x+k$ is divisible by $x+1$?

A) −5
B) −2
C) 2
D) 5

$P(-1)=0$ $-1-5+1+k=0$
$k-5=0$
$k=5$

4. What is the value of a when $g(x)=x^3+ax+b$ is divisible by $(x-1)(x-2)$?

A) −7
B) −4
C) 4
D) 8

$g(1)=0$ $1+a+b=0$ $a+b=-1$
$g(2)=0$ $8+2a+b=0$ $-\ 2a+b=-8$
$-a=7$
$a=-7$

Algebra 2 Name________________________________ ID: 1

Rational Root Theorem Extra Practice

Date________________ Period____

State the possible rational zeros for each function.

1) $f(x) = x^3 - 13x^2 + 33x - 22$

2) $f(x) = 3x^3 - 5x^2 - 10x + 16$

3) $f(x) = 3x^3 - 28x^2 + 13x + 22$

4) $f(x) = 6x^3 - 31x^2 + 38x + 5$

Find all roots.

5) $4x^3 - x^2 - 4x + 1 = 0$

6) $2x^3 - x^2 - x = 0$

7) $5x^4 + 52x^3 - 8x^2 - 15x = 0$

8) $4x^4 + 7x^3 + 2x^2 - x = 0$

Find all roots. One factor has been given.

9) $3x^6 - x^5 - 9x^4 + 9x^3 - 2x^2 = 0;\ x + 2$

10) $3x^6 + 14x^5 + 27x^4 + 56x^3 + 60x^2 = 0;\ x + 3$

Answers to Rational Root Theorem Extra Practice (ID: 1)

1) $\pm 1, \pm 2, \pm 11, \pm 22$

2) $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}, \pm \frac{16}{3}$

3) $\pm 1, \pm 2, \pm 11, \pm 22, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{11}{3}, \pm \frac{22}{3}$

4) $\pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}, \pm \frac{1}{3}, \pm \frac{5}{3}, \pm \frac{1}{6}, \pm \frac{5}{6}$

5) $\left\{-1, \frac{1}{4}, 1\right\}$

6) $\left\{0, 1, -\frac{1}{2}\right\}$

7) $\left\{0, \frac{3}{5}, \frac{-11 + \sqrt{101}}{2}, \frac{-11 - \sqrt{101}}{2}\right\}$

8) $\left\{0, -1 \text{ mult. } 2, \frac{1}{4}\right\}$

9) $\left\{0 \text{ mult. } 2, 1 \text{ mult. } 2, \frac{1}{3}, -2\right\}$

10) $\left\{0 \text{ mult. } 2, -\frac{5}{3}, 2i, -2i, -3\right\}$

Rational Roots Theorem Practice

For each of the following polynomials, list all possible roots, find the linear factorization and then list all roots.

1. $f(x) = x^3 - 8x^2 - 23x + 30$

Possible Roots:

Linear Factorization:

Roots:

2. $f(x) = x^4 - 6x^3 - 20x^2 - 42x - 189$

Possible Roots:

Linear Factorization:

Roots:

3. $f(x) = x^3 + x^2 - 2x - 2$

Possible Roots:

Linear Factorization:

Roots:

4. $f(x) = x^4 + x^3 + x^2 - 9x - 10$

Possible Roots:

Linear Factorization:

Roots:

Key 2017

Rational Roots Theorem Practice

For each of the following polynomials, list all possible roots, find the linear factorization and then list all roots.

1. $f(x) = x^3 - 8x^2 - 23x + 30$ Not Factorable

Possible Roots:

$p = 30 \rightarrow \pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30$

$q = 1 \rightarrow \pm1$

$\frac{p}{q} = \pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30$

$1 \,|\, 1 \;\; -8 \;\; -23 \;\; 30$; $\;\downarrow \;\; 1 \;\; -7 \;\; -30$; $\;1 \;\; -7 \;\; -30 \;\; |0R$

$x^2 - 7x - 30 = 0$

$(x-10)(x+3) = 0$

$x = 10, -3$

Linear Factorization: $(x-1)(x-10)(x+3)$

Roots: $x = 1, 10, -3$

2. $f(x) = x^4 - 6x^3 - 20x^2 - 42x - 189$ Not Factorable

Possible Roots:

$p = 189 \rightarrow \pm1, \pm3, \pm7, \pm9, \pm21, \pm27, \pm63, \pm189$

$q = 1 \rightarrow \pm1$

$\frac{p}{q} = \pm1, \pm3, \pm7, \pm9, \pm21, \pm27, \pm63, \pm189$

$-3 \,|\, 1 \;\; -6 \;\; -20 \;\; -42 \;\; -189$; $\;\downarrow \;\; -3 \;\; 27 \;\; -21 \;\; 189$; $\;1 \;\; -9 \;\; 7 \;\; -63 \;\; |0R$

$9 \,|\, 1 \;\; -9 \;\; 7 \;\; -63$; $\;\downarrow \;\; 9 \;\; 0 \;\; 63$; $\;1 \;\; 0 \;\; 7 \;\; |0R$

$x^2 + 7 = 0$

$\sqrt{x^2} = \sqrt{-7}$ $\quad x = \pm i\sqrt{7}$

Linear Factorization: $(x+3)(x-9)(x+i\sqrt{7})(x-i\sqrt{7})$

Roots: $x = -3, 9, \pm i\sqrt{7}$

3. $f(x) = x^3 + x^2 - 2x - 2$ Factorable

Possible Roots:

$p = 2 \rightarrow \pm1, \pm2$

$q = 1 \rightarrow \pm1$

$\frac{p}{q} = \pm1, \pm2$

$(x^3 + x^2)(-2x - 2) = 0$

$x^2(x+1) - 2(x+1) = 0$

$(x+1)(x^2 - 2) = 0$

$(x+1)(x+\sqrt{2})(x-\sqrt{2}) = 0$

$x = -1, \pm\sqrt{2}$

Linear Factorization: $(x+1)(x+\sqrt{2})(x-\sqrt{2})$

Roots: $x = -1, \pm\sqrt{2}$

4. $f(x) = x^4 + x^3 + x^2 - 9x - 10$ Not Factorable

Possible Roots:

$p = 10 \rightarrow \pm1, \pm2, \pm5, \pm10$

$q = 1 \rightarrow \pm1$

$\frac{p}{q} = \pm1, \pm2, \pm5, \pm10$

$2 \,|\, 1 \;\; 1 \;\; 1 \;\; -9 \;\; -10$; $\;\downarrow \;\; 2 \;\; 6 \;\; 14 \;\; 10$; $\;1 \;\; 3 \;\; 7 \;\; 5 \;\; |0R$

$-1 \,|\, 1 \;\; 3 \;\; 7 \;\; 5$; $\;\downarrow \;\; -1 \;\; -2 \;\; -5$; $\;1 \;\; 2 \;\; 5 \;\; |0R$

$x^2 + 2x + 5 = 0 \rightarrow$

$x = \frac{-2 \pm \sqrt{4 - 4(1)(5)}}{2}$

$x = \frac{-2 \pm \sqrt{-16}}{2}$

$x = \frac{-2 \pm 4i}{2}$

$x = -1 \pm 2i$

Linear Factorization: $(x-2)(x+1)(x+1+2i)(x+1-2i)$

Roots: $x = 2, -1, -1 \pm 2i$

5. $f(x) = x^5 - x^4 - 7x^3 + 11x^2 - 8x + 12$

Possible Roots:

Linear Factorization:

Roots:

6. $f(x) = x^5 + 3x^4 - 22x^3 - 83x^2 - 39x + 20$

Possible Roots:

Linear Factorization:

Roots:

7. $f(x) = x^4 + 41x^2 + 400$

Possible Roots:

Linear Factorization:

Roots:

8. $f(x) = 2x^5 + x^4 - 32x - 16$

Possible Roots:

Linear Factorization:

Roots:

Key 2017

5. $f(x) = x^5 - x^4 - 7x^3 + 11x^2 - 8x + 12$ Not Factorable

Possible Roots:

$p = 12 \rightarrow \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$q = 1 \rightarrow \pm1$

$\frac{p}{q} = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

2	1	-1	-7	11	-8	12
	↓	2	2	-10	2	-12
	1	1	-5	1	-6	0 R

-3	1	1	-5	1	-6
	↓	-3	6	-3	6
	1	-2	1	-2	0 R

$(x^3 - 2x^2)(+x - 2) = 0$

$x^2(x-2)\,1(x-2) = 0$

$(x-2)(x^2+1) = 0$

Linear Factorization:

$(x-2)(x+i)(x-i)$

Roots:

$x = 2, \pm i$

6. $f(x) = x^5 + 3x^4 - 22x^3 - 83x^2 - 39x + 20$ Not Factorable

Possible Roots:

$p = 20 \rightarrow \pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

$q = 1 \rightarrow \pm1$

$\frac{p}{q} = \pm1, \pm2, \pm4, \pm5, \pm10, \pm20$

-1	1	3	-22	-83	-39	20
	↓	-1	-2	24	59	-20
	1	2	-24	-59	20	0 R

-4	1	2	-24	-59	20
	↓	-4	8	64	-20
	1	-2	-16	5	0 R

5	1	-2	-16	5
	↓	5	15	-5
	1	3	-1	0 R

$x^2 + 3x - 1 = 0$

$x = \frac{-3 \pm \sqrt{9-4(1)(-1)}}{2}$

$x = \frac{-3 \pm \sqrt{13}}{2}$

Linear Factorization:

$(x+1)(x+4)(x-5)\left(x + \frac{3 \pm \sqrt{13}}{2}\right)\left(x + \frac{3 - \sqrt{13}}{2}\right)$

Roots:

$x = -1, -4, 5, \frac{-3 \pm \sqrt{13}}{2}$

7. $f(x) = x^4 + 41x^2 + 400$ Factorable

Possible Roots:

$\frac{p}{q}$ $p = 400 \rightarrow \pm1, \pm2, \pm4, \pm5, \pm8, \pm10, \pm16, \pm20, \pm25, \pm40, \pm50, \pm80, \pm100, \pm200, \pm400$

$(x^2 + 25)(x^2 + 16) = 0$

$(x+5i)(x-5i)(x+4i)(x-4i) = 0$

$x = \pm5i, \pm4i$

Linear Factorization:

$(x+5i)(x-5i)(x+4i)(x-4i)$

Roots:

$x = \pm5i, \pm4i$

8. $f(x) = 2x^5 + x^4 - 32x - 16$ Factorable

Possible Roots:

$p = 16 \rightarrow \pm1, \pm2, \pm4, \pm8, \pm16$

$q = 2 \rightarrow \pm1, \pm2$

$\frac{p}{q} = \pm1, \pm\frac{1}{2}, \pm2, \pm4, \pm8, \pm16$

$(2x^5 + x^4)(-32x - 16) = 0$

$x^4(2x+1) - 16(2x+1) = 0$

$(x^4 - 16)(2x+1) = 0$

$(x^2+4)(x^2-4)(2x+1) = 0$

$(x+2i)(x-2i)(x+2)(x-2)(2x+1) = 0$

$x = \pm2i, \pm2, -\frac{1}{2}$

Linear Factorization:

$(x+2i)(x-2i)(x+2)(x-2)(2x+1)$

Roots:

$x = \pm2i, \pm2, -\frac{1}{2}$

11) $f(x) = 5x^3 + 9x^2 - 26x - 24$;

12) $f(x) = 6x^3 + 7x^2 - 1$

Factor each and find all zeros. One zero has been given.

13) $f(x) = 5x^3 + 4x^2 - 20x - 16$

14) $f(x) = 25x^4 - 40x^3 - 19x^2 - 2x$

15) $f(x) = 3x^4 + 5x^3 + 81x + 135$

16) $f(x) = 2x^4 - x^3 - 18x^2 + 9x$

17) $f(x) = 10x^3 - 41x^2 + 32x + 20$

18) $f(x) = 3x^3 + 4x^2 - 35x - 12$

11) $f(x) = 5x^3 + 9x^2 - 26x - 24;\ x + 3$

Factors to: $f(x) = (5x + 4)(x - 2)(x + 3)$

Zeros: $\left\{-\frac{4}{5}, 2, -3\right\}$

12) $f(x) = 6x^3 + 7x^2 - 1;\ 2x + 1$

Factors to: $f(x) = (3x - 1)(x + 1)(2x + 1)$

Zeros: $\left\{\frac{1}{3}, -1, -\frac{1}{2}\right\}$

Factor each and find all zeros. One zero has been given.

13) $f(x) = 5x^3 + 4x^2 - 20x - 16;\ 2$

Factors to: $f(x) = (5x + 4)(x + 2)(x - 2)$

Zeros: $\left\{-\frac{4}{5}, -2, 2\right\}$

14) $f(x) = 25x^4 - 40x^3 - 19x^2 - 2x;\ -\frac{1}{5}$

Factors to: $f(x) = x(5x + 1)^2(x - 2)$

Zeros: $\left\{0, -\frac{1}{5} \text{ mult. } 2, 2\right\}$

15) $f(x) = 3x^4 + 5x^3 + 81x + 135;\ -\frac{5}{3}$

Factors to: $f(x) = (x + 3)(x^2 - 3x + 9)(3x + 5)$

Zeros: $\left\{-3, \frac{3 + 3i\sqrt{3}}{2}, \frac{3 - 3i\sqrt{3}}{2}, -\frac{5}{3}\right\}$

16) $f(x) = 2x^4 - x^3 - 18x^2 + 9x;\ -3$

Factors to: $f(x) = x(2x - 1)(x - 3)(x + 3)$

Zeros: $\left\{0, \frac{1}{2}, 3, -3\right\}$

17) $f(x) = 10x^3 - 41x^2 + 32x + 20;\ \frac{5}{2}$

Factors to: $f(x) = (5x + 2)(x - 2)(2x - 5)$

Zeros: $\left\{-\frac{2}{5}, 2, \frac{5}{2}\right\}$

18) $f(x) = 3x^3 + 4x^2 - 35x - 12;\ 3$

Factors to: $f(x) = (3x + 1)(x + 4)(x - 3)$

Zeros: $\left\{-\frac{1}{3}, -4, 3\right\}$