ONLINE WORKSHEET PACKAGE

AP CALCULUS

DERIVATIVE RULES

POWER FUNCTION DERIVATIVES

DR AHN MATH & LEARNING CENTER

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< Special Function Derivatives >

$f(x)$	$f'(x)$
x^n	$n x^{n-1}$
e^x	e^x
a^x	$a^x \cdot \ln a$
$\ln x$	$\frac{1}{x}$
$\log_a x$	$\frac{1}{x} \cdot \frac{1}{\ln a}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$

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< Power Function Derivatives >

$y = x^n \rightarrow y' = nx^{n-1}$

$y = c \rightarrow y' = 0$

$y' = f'(x) = \frac{dy}{dx} = \frac{d}{dx} f(x)$

$y = x^4 \rightarrow y' = 4x^3$

$y = 2x \rightarrow y' = 2$

$y = \sqrt{x} \rightarrow y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$y = x^{-4} \rightarrow y' = -4x^{-5}$

$y = -2 \rightarrow y' = 0$

$y = \frac{1}{x} \rightarrow y' = -x^{-2} = -\frac{1}{x^2}$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$ $\quad$ $(\frac{1}{x})' = -\frac{1}{x^2}$

$y = -3x^4 + 2x^3 + 3x - 4 \rightarrow$

$y = \frac{2}{x} - \frac{3}{x^2} + \frac{4}{x^4} \rightarrow$

$y = \frac{2x^5 - 3x^3 + 2x - 1}{x}$

$y = \frac{\sqrt{x} + x^{\frac{2}{3}} - 3x^{\frac{1}{5}}}{x}$

< Power Function Derivatives >

$y = x^n \rightarrow y' = nx^{n-1}$

$y = c \rightarrow y' = 0$

$y' = f'(x) = \frac{dy}{dx} = \frac{d}{dx} f(x)$

$y = x^4 \rightarrow y' = 4x^3$ $\quad$ $y = x^{-4} \rightarrow y' = -4x^{-5}$

$y = 2x \rightarrow y' = 2$ $\quad$ $y = -2 \rightarrow y' = 0$

$y = \sqrt{x} \rightarrow y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ $\quad$ $y = \frac{1}{x} \rightarrow y' = -x^{-2} = -\frac{1}{x^2}$

$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$ $\quad$ $(\frac{1}{x})' = -\frac{1}{x^2}$

$y = -3x^4 + 2x^3 + 3x - 4 \rightarrow y' = -12x^3 + 6x^2 + 3$

$y = \frac{2}{x} - \frac{3}{x^2} + \frac{4}{x^4} \rightarrow y' = -\frac{2}{x^2} + \frac{6}{x^3} - \frac{16}{x^5}$

$y = \frac{2x^5 - 3x^3 + 2x - 1}{x}$ $\left(2x^4 - 3x^2 + 2 - \frac{1}{x}\right) \rightarrow y' = 8x^3 - 6x + \frac{1}{x^2}$

$= 2x^4 - 3x^2 + 2 - \frac{1}{x} \rightarrow y' = 8x^3 - 6x + \frac{1}{x^2}$

$y = \frac{\sqrt{x} + x^{\frac{2}{3}} - 3x^{\frac{4}{5}}}{x}$ $\left(x^{-\frac{1}{2}} - x^{-\frac{1}{3}} - 3x^{-\frac{4}{5}}\right) \rightarrow y' = -\frac{1}{2}x^{-\frac{3}{2}} + \frac{1}{3}x^{-\frac{4}{3}} + \frac{12}{5}x^{-\frac{9}{5}}$

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$$y = \frac{2}{5}x^4 - \frac{6}{7}x^3 + \frac{2}{3}x^2$$

$$y = \frac{2}{3}x^{\frac{3}{2}} - \frac{11}{7}x^{\frac{1}{2}} + \frac{1}{3}x^{-\frac{1}{3}} + x^{-\frac{5}{2}}$$

$$y = \frac{10x^{-4} + 2x^{-5} - 11x^{-13}}{2x^{-3}}$$

$$y = \frac{x^{\frac{5}{3}} + x^{\frac{11}{2}} - 3}{\sqrt[3]{x}}$$

$$y = \frac{2}{5}x^4 - \frac{6}{7}x^3 + \frac{2}{3}x^2$$

$$\rightarrow y' = \frac{8}{5}x^3 - \frac{18}{7}x^2 + \frac{4}{3}x$$

$$y = \frac{2}{3}x^{\frac{3}{2}} - \frac{11}{7}x^{\frac{1}{2}} + \frac{1}{3}x^{-\frac{1}{3}} + x^{-\frac{5}{2}}$$

$$\rightarrow y' = x^{\frac{1}{2}} - \frac{11}{14}x^{-\frac{1}{2}} - \frac{1}{9}x^{-\frac{4}{3}} - \frac{5}{2}x^{-\frac{7}{2}}$$

$$y = \frac{10x^{-4} + 2x^{-5} - 11x^{-13}}{2x^{-3}} = 5x^{-1} - x^{-2} - \frac{11}{2}x^{-10}$$

$$\rightarrow y' = -5x^{-2} + 2x^{-3} + 55x^{-11}$$

$$y = \frac{x^{\frac{5}{3}} + x^{\frac{11}{2}} - 3}{\sqrt[3]{x}} = x^{\frac{4}{3}} + x^{\frac{31}{6}} - 3x^{-\frac{1}{3}}$$

$$\rightarrow y' = \frac{4}{3}x^{\frac{1}{3}} + \frac{31}{6}x^{\frac{25}{6}} + x^{-\frac{4}{3}}$$

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< Product Rule >

$$(fg)' = f'g + fg'$$

$y = (2x+1)(x^3-2x)$ →

< Quotient Rule >

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$y = \dfrac{x^2+2x}{x+1}$ →

$y = (x^2+2x)(x+1)^{-1}$ →

$y = \dfrac{1}{x+1}$ →

< Product Rule >

$$(fg)' = f'g + fg'$$

$y = (2x+1)(x^3-2x) \rightarrow y' = 2(x^3-2x) + (2x+1)(3x^2-2)$

$= 2x^3 - 4x + 6x^3 - 4x + 3x^2 - 2$

$= 8x^3 + 3x^2 - 8x - 2$

$y = 2x^4 + x^3 - 4x^2 - 2x \rightarrow y' = 8x^3 + 3x^2 - 8x - 2$

< Quotient Rule >

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$y = \dfrac{x^2+2x}{x+1} \rightarrow y' = \dfrac{(2x+2)(x+1) - (x^2+2x)}{(x+1)^2}$

$= \dfrac{2x^2+4x+2-x^2-2x}{(x+1)^2} = \dfrac{x^2+2x+2}{(x+1)^2}$

$y = (x^2+2x)(x+1)^{-1} \rightarrow y' = (2x+2)(x+1)^{-1} - (x^2+2x)(x+1)^{-2}$

$= (x+1)^{-2}\left((2x+2)(x+1) - (x^2+2x)\right) = \dfrac{x^2+2x+2}{(x+1)^2}$

$y = \dfrac{1}{x+1} \rightarrow y' = \dfrac{-1}{(x+1)^2}$ $\quad y = (x+1)^{-1} \rightarrow y' = -(x+1)^{-2}$

$= -\dfrac{1}{(x+1)^2}$

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< Chain Rule >

$$(f(g))' = f'(g) \cdot g'$$

$y = (2x+1)^3$ →

$y = (x^2-4)^5$ →

$y = (2x+1)^3(x^2-1)^2$ →

$$y = \frac{(x^2+1)^2}{(2x-1)^3} \rightarrow$$

< Chain Rule >

$$(f(g))' = f'(g) \cdot g'$$

$$y = (2x+1)^3 \rightarrow y' = 3(2x+1)^2 \cdot 2 = 6(2x+1)^2$$

$$y = (x^2-4)^5 \rightarrow y' = 5(x^2-4)^4 \cdot 2x = 10x(x^2-4)^4$$

$$y = (2x+1)^3(x^2-1)^2 \rightarrow y' = 3(2x+1)^2 \cdot 2(x^2-1)^2 + (2x+1)^3 \cdot 2(x^2-1) \cdot 2x$$

$$= 2(2x+1)^2(x^2-1)(3(x^2-1) + 2x(2x+1))$$

$$= 2(2x+1)^2(x^2-1)(7x^2+2x-3)$$

$$y = \frac{(x^2+1)^2}{(2x-1)^3} \rightarrow y' = \frac{2(x^2+1)2x(2x-1)^3 - (x^2+1)^2 3(2x-1)^2 \cdot 2}{(2x-1)^6}$$

$$= \frac{2(x^2+1)(2x-1)^2(2x(2x-1) - 3(x^2+1))}{(2x-1)^6}$$

$$= \frac{2(x^2+1)(2x-1)^2(x^2-2x-3)}{(2x-1)^6}$$

$$= \frac{2(x^2+1)(x-3)(x+1)}{(2x-1)^4}$$

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- $y = (2x^2+1)^3(4x-1)^4$

- $y = (2x-3)^{-3}(2-3x)^{-4}$

- $y = \sqrt[4]{x^2-1}\,\sqrt{x^3-1}$

- $y = (2x-1)^{\frac{1}{3}}(2-3x^2)^{\frac{1}{2}}$

- $y = (2x^2+1)^3(4x-1)^4$

$\rightarrow y' = 3(2x^2+1)^2 \cdot 4x(4x-1)^4 + (2x^2+1)^3 4(4x-1)^3 \cdot 4$

$= 4(2x^2+1)^2(4x-1)^3(3x(4x-1) + 4(2x^2+1))$

$= 4(2x^2+1)^2(4x-1)^3(20x^2-3x+4)$

- $y = (2x-3)^{-3}(2-3x)^{-4}$

$\rightarrow y' = -3(2x-3)^{-4} \cdot 2(2-3x)^{-4} + (2x-3)^{-3} \cdot (-4)(2-3x)^{-5} \cdot (-3)$

$= -6(2x-3)^{-4}(2-3x)^{-5}((2-3x) - 2(2x-3))$

$= -6(2x-3)^{-4}(2-3x)^{-5}(8-7x)$

- $y = \sqrt[4]{x^2-1}\sqrt{x^3-1} = (x^2-1)^{\frac{1}{4}}(x^3-1)^{\frac{1}{2}}$

$\rightarrow y' = \frac{1}{4}(x^2-1)^{-\frac{3}{4}} \cdot 2x(x^3-1)^{\frac{1}{2}} + (x^2-1)^{\frac{1}{4}} \cdot \frac{1}{2}(x^3-1)^{-\frac{1}{2}} \cdot 3x^2$

$= \frac{1}{2}x(x^2-1)^{-\frac{3}{4}} \cdot (x^3-1)^{-\frac{1}{2}}(x^3-1 + 3x(x^2-1))$

$= \frac{1}{2}x(x^2-1)^{-\frac{3}{4}}(x^3-1)^{-\frac{1}{2}}(4x^3-3x-1) = \frac{x(4x^3-3x-1)}{(x^2-1)^{\frac{3}{4}}(x^3-1)^{\frac{1}{2}}}$

- $y = (2x-1)^{\frac{1}{3}}(2-3x^2)^{\frac{1}{4}}$

$y' = \frac{1}{3}(2x-1)^{-\frac{2}{3}} \cdot 2(2-3x^2)^{\frac{1}{4}} + (2x-1)^{\frac{1}{3}} \cdot \frac{1}{4}(2-3x^2)^{-\frac{3}{4}} \cdot (-6x)$

$= \frac{1}{6}(2x-1)^{-\frac{2}{3}}(2-3x^2)^{-\frac{3}{4}}(4(2-3x^2) - 9x(2x-1))$

$= \frac{1}{6}(2x-1)^{-\frac{2}{3}}(2-3x^2)^{-\frac{3}{4}}(-30x^2+9x+8)$

$= -\frac{1}{6}\frac{30x^2-9x-8}{(2x-1)^{\frac{2}{3}}(2-3x^2)^{\frac{3}{4}}}$

Differentiation - Power, Constant, and Sum Rules

Date_____ Period____

Differentiate each function with respect to *x*.

1) $y = 5$

2) $f(x) = 5x^{18}$

3) $y = 4x^5 + x$

4) $f(x) = 4x^4 - 5x - 3$

5) $y = 3x^{\frac{5}{4}}$

6) $y = \frac{5}{4}x^{\frac{2}{3}}$

7) $y = -4x^{-5}$

8) $y = \frac{3}{x^3}$

9) $y = x^{\frac{2}{3}}$

10) $f(x) = -2\sqrt[4]{x}$

Kuta Software - Infinite Calculus Name_____

Differentiation - Power, Constant, and Sum Rules

Date_____ Period____

Differentiate each function with respect to *x*.

1) $y = 5$

$$\frac{dy}{dx} = 0$$

2) $f(x) = 5x^{18}$

$$f'(x) = 90x^{17}$$

3) $y = 4x^5 + x$

$$\frac{dy}{dx} = 20x^4 + 1$$

4) $f(x) = 4x^4 - 5x - 3$

$$f'(x) = 16x^3 - 5$$

5) $y = 3x^{\frac{5}{4}}$

$$\frac{dy}{dx} = \frac{15x^{\frac{1}{4}}}{4}$$

6) $y = \frac{5}{4}x^{\frac{2}{3}}$

$$\frac{dy}{dx} = \frac{5}{6}x^{-\frac{1}{3}}$$
$$= \frac{5}{6x^{\frac{1}{3}}}$$

7) $y = -4x^{-5}$

$$\frac{dy}{dx} = 20x^{-6}$$
$$= \frac{20}{x^6}$$

8) $y = \frac{3}{x^3}$

$$\frac{dy}{dx} = -9x^{-4}$$
$$= -\frac{9}{x^4}$$

9) $y = x^{\frac{2}{3}}$

$$\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}}$$
$$= \frac{2}{3x^{\frac{1}{3}}}$$

10) $f(x) = -2\sqrt[4]{x}$

$$f'(x) = -\frac{1}{2}x^{-\frac{3}{4}}$$
$$= -\frac{1}{2x^{\frac{3}{4}}}$$

11) $y = \frac{2}{3}x^4 + 5x - x^{-3}$

12) $y = -\frac{1}{2}x^4 + 3x^{\frac{5}{3}} + 2x$

Differentiate each function with respect to the given variable.

13) $y = -3r^5 - 5r^2$

14) $f(s) = -\frac{3}{s^2} - \frac{4}{s^4}$

15) $f(x) = \frac{2}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{3}{5}}$

16) $h(s) = \sqrt{2} \cdot \sqrt[3]{s} + \sqrt{2} \cdot \sqrt[5]{s}$

Differentiate each function with respect to *x*. Problems may contain constants a, b, and c.

17) $y = 5c$

18) $y = 4ax^{3a} - bx^{3c}$

11) $y = \frac{2}{3}x^4 + 5x - x^{-3}$

$\frac{dy}{dx} = \frac{8}{3}x^3 + 5 + 3x^{-4}$

$= \frac{8x^3}{3} + 5 + \frac{3}{x^4}$

12) $y = -\frac{1}{2}x^4 + 3x^{\frac{5}{3}} + 2x$

$\frac{dy}{dx} = -2x^3 + 5x^{\frac{2}{3}} + 2$

Differentiate each function with respect to the given variable.

13) $y = -3r^5 - 5r^2$

$\frac{dy}{dr} = -15r^4 - 10r$

14) $f(s) = -\frac{3}{s^2} - \frac{4}{s^4}$

$f'(s) = 6s^{-3} + 16s^{-5}$

$= \frac{6}{s^3} + \frac{16}{s^5}$

15) $f(x) = \frac{2}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{3}{5}}$

$f'(x) = x^{\frac{1}{2}} - \frac{9}{20}x^{-\frac{2}{5}}$

$= x^{\frac{1}{2}} - \frac{9}{20x^{\frac{2}{5}}}$

16) $h(s) = \sqrt{2} \cdot \sqrt[3]{s} + \sqrt{2} \cdot \sqrt[5]{s}$

$h'(s) = \frac{1}{3}s^{-\frac{2}{3}}\sqrt{2} + \frac{1}{5}s^{-\frac{4}{5}}\sqrt{2}$

$= \frac{\sqrt{2}}{3s^{\frac{2}{3}}} + \frac{\sqrt{2}}{5s^{\frac{4}{5}}}$

Differentiate each function with respect to *x*. Problems may contain constants a, b, and c.

17) $y = 5c$

$\frac{dy}{dx} = 0$

18) $y = 4ax^{3a} - bx^{3c}$

$\frac{dy}{dx} = 12a^2x^{3a-1} - 3bcx^{3c-1}$

Student Name: _____ **Score:**

Derivatives using Power Rule

Find the derivatives using power rule:

$y = 10x^3$

$y = \frac{1}{2}x^{-2}$

$y = \frac{1}{2\sqrt{x}}$

$y = 3x^{\frac{-1}{15}}$

$y = 8x^6 + 2x^{17}$

$y = \sqrt[5]{x}$

$y = x^{\frac{1}{31}} + x^{\frac{-1}{7}}$

$y = 2x^{12} + 6x^7 + x^4$

$y = \frac{5}{3}x^3 - \frac{7}{6}x^6 + \frac{6}{4}x^8$

$y = \frac{1}{2}x^{\frac{3}{2}} - \frac{22}{7}x^{\frac{-5}{2}} + x^{\frac{3}{7}}$

Student Name: _____ **Score:**

Answers:

$\frac{dy}{dx} = 30x^2$

$\frac{dy}{dx} = \frac{-1}{x^3}$

$\frac{dy}{dx} = \frac{-1}{4}\, x^{\frac{-3}{2}}$

$\frac{dy}{dx} = -\frac{1}{5}\, x^{\frac{-16}{15}}$

$\frac{dy}{dx} = 48x^5 + 34x^{16}$

$\frac{dy}{dx} = \frac{1}{5}\, x^{\frac{-4}{5}}$

$\frac{dy}{dx} = \frac{1}{31}\, x^{\frac{-30}{31}} - \frac{1}{7}\, x^{\frac{-8}{7}}$

$\frac{dy}{dx} = 24x^{11} + 42x^6 + 4x^3$

$\frac{dy}{dx} = 5\, x^2 - 7\, x^5 + 12\, x^7$

$\frac{dy}{dx} = \frac{3}{4}\, x^{\frac{1}{2}} + \frac{55}{7}\, x^{\frac{-7}{2}} + \frac{3}{7} x^{\frac{-4}{7}}$

Student Name: _____ **Score:**

Derivatives using Power Rule

Find the derivatives using power rule:

$$y = \frac{8x^5 + 4x^4}{2x^2}$$

$$y = \frac{15x^7 + 21x^5 + 12x^3}{3x}$$

$$y = \frac{-22x^{-5} - 17x^{-11}}{21x^{-4}}$$

$$y = \frac{2x^{\frac{11}{3}} + 4x^{\frac{5}{4}} - 3x^{\frac{7}{2}}}{4x^{\frac{2}{3}}}$$

$$y = \frac{7x^2 + 5x^9}{4x^7}$$

$$y = \frac{\sqrt{x} - \sqrt[3]{x}}{\sqrt[5]{x}}$$

$$y = \frac{5x^{-45} + 15x^{-4} - 5x^{-17}}{5x^{-2}}$$

$$y = \frac{5x^2 + 12x^{-5}}{\sqrt{x}}$$

$$y = \frac{\frac{2}{7}x^{\frac{-5}{11}} + \frac{16}{7}x^{\frac{-12}{11}}}{x^{\frac{-21}{11}}}$$

$$y = \frac{x^{\frac{7}{3}} + x^{\frac{10}{3}}}{\sqrt[3]{x}}$$

Student Name: _____ **Score:**

Answers:

$\frac{dy}{dx} = 12x^2 + 4x$

$\frac{dy}{dx} = 30x^5 + 28x^3 + 8x$

$\frac{dy}{dx} = \frac{22}{21x^2} + \frac{119}{21x^8}$

$\frac{dy}{dx} = \frac{3}{2}x^2 + \frac{7}{12}x^{-\frac{5}{12}} - \frac{17}{8}x^{\frac{11}{6}}$

$\frac{dy}{dx} = \frac{-35}{4x^6} + \frac{10x}{4}$

$\frac{dy}{dx} = \frac{3}{10}x^{-\frac{7}{10}} - \frac{2}{15}x^{-\frac{13}{15}}$

$\frac{dy}{dx} = \frac{-43}{x^{44}} - \frac{6}{x^3} + \frac{75}{x^{16}}$

$\frac{dy}{dx} = \frac{15}{2}\sqrt{x} - \frac{66}{\sqrt{x^{13}}}$

$\frac{dy}{dx} = \frac{32}{77}x^{\frac{5}{11}} + \frac{144}{77}x^{-\frac{2}{11}}$

$\frac{dy}{dx} = 2x + 3x^2$

AP Calculus AB **Name** _____

Rules for Differentiation

Differentiate the following functions (with respect to the independent variable).

1. $f(x) = 186.5$
2. $y = 5x - 1$
3. $f(x) = x^2 + 3x - 4$
4. $y = -4x^{10}$
5. $f(x) = \sqrt{30}$
6. $y = 5x^8 - 2x^5 + 6$
7. $f(t) = \frac{1}{4}(t^4 + 8)$
8. $f(t) = \frac{1}{2}t^6 - 3t^4 + t$
9. $y = x^{-\frac{2}{5}}$
10. $V(r) = \frac{4}{3}\pi r^3$
11. $R(t) = 5t^{-\frac{3}{5}}$
12. $R(x) = \frac{\sqrt{10}}{x^7}$
13. $y = \sqrt{x} - \sqrt[3]{x}$
14. $g(x) = x^2 + \frac{1}{x^2}$
15. $f(x) = ax^2 + bx + c$
16. $y = 4x^7$
17. $f(x) = \sqrt{2}x + \frac{1}{\sqrt{2}}$
18. $y = x^{-3} + \frac{1}{x^7}$
19. $y = \pi^3$
20. $g(x) = -\frac{1}{3}(x^7 + 2x - 9)$
21. $y = \frac{x^2 + 1}{5}$
22. $f(x) = -3x^{-8} + 2\sqrt{x}$
23. $y = -3x^{12}$
24. $y = \frac{1}{a}\left(x^2 + \frac{1}{b}x + c\right)$

AP Calculus AB Name Key

Rules for Differentiation

Differentiate the following functions (with respect to the independent variable).

1. $f(x) = 186.5$

$f'(x) = 0$

2. $y = 5x - 1$ (Linear)

$\frac{dy}{dx} = 5$

3. $f(x) = x^2 + 3x - 4$

$f'(x) = 2x + 3$

4. $y = -4x^{10}$ (-1)

$y' = -40x^9$

5. $f(x) = \sqrt{30}$

$f'(x) = 0$

6. $y = 5x^8 - 2x^5 + 6$ (-1, -1)

$\frac{dy}{dx} = 40x^7 - 10x^4$

Constant Multiple Rule

7. $f(t) = \frac{1}{4}(t^4 + 8)$

$f'(t) = \frac{1}{4} \cdot 4t^3$

$f'(t) = t^3$

8. $f(t) = \frac{1}{2}t^6 - 3t^4 + t$ (-1, -1, Linear)

$f'(t) = 3t^5 - 12t^3 + 1$

9. $y = x^{-\frac{2}{5}}$ (-1)

$\frac{dy}{dx} = -\frac{2}{5}x^{-7/5}$

Constant Multiple

10. $V(r) = \frac{4}{3}\pi r^3 = \frac{4\pi}{3} \cdot r^3$

$V'(r) = \frac{4\pi}{3} \cdot 3r^2$

$V'(r) = 4\pi r^2$

11. $R(t) = 5t^{-\frac{3}{5}}$ (-1)

$R'(t) = -3t^{-8/5}$

12. $R(x) = \frac{\sqrt{10}}{x^7} = \sqrt{10} \cdot x^{-7}$ (-7, -1)

$R'(x) = -7\sqrt{10}\,x^{-8} = \frac{-7\sqrt{10}}{x^8}$

13. $y = \sqrt{x} - \sqrt[3]{x}$

$y = x^{1/2} - x^{1/3}$ (-1, -1)

$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{3}x^{-2/3}$

$= \frac{1}{2\sqrt{x}} - \frac{1}{3\sqrt[3]{x^2}}$

14. $g(x) = x^2 + \frac{1}{x^2}$

$g(x) = x^2 + x^{-2}$ (-1)

$g'(x) = 2x - 2x^{-3}$

$g'(x) = 2x - \frac{2}{x^3}$

15. $f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

16. $y = 4x^7$

$\frac{dy}{dx} = 28x^6$

17. $f(x) = \sqrt{2}x + \frac{1}{\sqrt{2}}$ (Linear, constant)

$f'(x) = \sqrt{2}$

18. $y = x^{-3} + \frac{1}{x^7} = x^{-3} + x^{-7}$ (-1, -1)

$\frac{dy}{dx} = -3x^{-4} - 7x^{-8}$

$\frac{dy}{dx} = \frac{-3}{x^4} - \frac{7}{x^8}$

19. $y = \pi^3$ (constant)

$\frac{dy}{dx} = 0$

Constant Multiple

20. $g(x) = -\frac{1}{3}(x^7 + 2x - 9)$

$g'(x) = -\frac{1}{3} \cdot [7x^6 + 2]$

or

$g'(x) = -\frac{7}{3}x^6 - \frac{2}{3}$

21. $y = \frac{x^2 + 1}{5} \Rightarrow y = \frac{x^2}{5} + \frac{1}{5}$

$y = \frac{1}{5}x^2 + \frac{1}{5}$

$y' = \frac{2}{5}x$

22. $f(x) = -3x^{-8} + 2\sqrt{x}$

$f(x) = -3x^{-8} + 2x^{1/2}$ (-1, -1)

$f'(x) = 24x^{-9} + x^{-1/2}$

$f'(x) = \frac{24}{x^9} + \frac{1}{\sqrt{x}}$

23. $y = -3x^{12}$ (-1)

$y' = -36x^{11}$

24. $y = \frac{1}{a}\left(x^2 + \frac{1}{b}x + c\right)$ (Constant Multiple)

$y' = \frac{1}{a} \cdot \left[2x + \frac{1}{b}\right]$

or

$y' = \frac{2x}{a} + \frac{1}{ab}$

Name_____

Differentiation - Product Rule

Date_____ Period____

Differentiate each function with respect to *x*.

1) $y = -x^3(3x^4 - 2)$

2) $f(x) = x^2(-3x^2 - 2)$

3) $y = (-2x^4 - 3)(-2x^2 + 1)$

4) $f(x) = (2x^4 - 3)(x^2 + 1)$

5) $f(x) = (5x^5 + 5)(-2x^5 - 3)$

6) $f(x) = \left(-3 + x^{-3}\right)(-4x^3 + 3)$

7) $y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$

8) $y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$

Kuta Software - Infinite Calculus Name_____

Differentiation - Product Rule

Date_____ Period____

Differentiate each function with respect to x.

1) $y = -x^3(3x^4 - 2)$

$$\frac{dy}{dx} = -x^3 \cdot 12x^3 + (3x^4 - 2) \cdot -3x^2$$
$$= -21x^6 + 6x^2$$

2) $f(x) = x^2(-3x^2 - 2)$

$$f'(x) = x^2 \cdot -6x + (-3x^2 - 2) \cdot 2x$$
$$= -12x^3 - 4x$$

3) $y = (-2x^4 - 3)(-2x^2 + 1)$

$$\frac{dy}{dx} = (-2x^4 - 3) \cdot -4x + (-2x^2 + 1) \cdot -8x^3$$
$$= 24x^5 - 8x^3 + 12x$$

4) $f(x) = (2x^4 - 3)(x^2 + 1)$

$$f'(x) = (2x^4 - 3) \cdot 2x + (x^2 + 1) \cdot 8x^3$$
$$= 12x^5 + 8x^3 - 6x$$

5) $f(x) = (5x^5 + 5)(-2x^5 - 3)$

$$f'(x) = (5x^5 + 5) \cdot -10x^4 + (-2x^5 - 3) \cdot 25x^4$$
$$= -100x^9 - 125x^4$$

6) $f(x) = (-3 + x^{-3})(-4x^3 + 3)$

$$f'(x) = (-3 + x^{-3}) \cdot -12x^2 + (-4x^3 + 3) \cdot -3x^{-4}$$
$$= 36x^2 - \frac{9}{x^4}$$

7) $y = (-2x^4 + 5x^2 + 4)(-3x^2 + 2)$

$$\frac{dy}{dx} = (-2x^4 + 5x^2 + 4) \cdot -6x + (-3x^2 + 2)(-8x^3 + 10x)$$
$$= 36x^5 - 76x^3 - 4x$$

8) $y = (x^4 + 3)(-4x^5 + 5x^4 + 5)$

$$\frac{dy}{dx} = (x^4 + 3)(-20x^4 + 20x^3) + (-4x^5 + 5x^4 + 5) \cdot 4x^3$$
$$= -36x^8 + 40x^7 - 60x^4 + 80x^3$$

9) $y = (5x^4 - 3x^2 - 1)(-5x^2 + 3)$

10) $f(x) = \left(-10x^2 - 7\sqrt[5]{x^2} + 9\right)(2x^3 + 4)$

11) $y = (5 + 3x^{-2})(4x^5 + 6x^3 + 10)$

12) $y = (-6x^4 + 2 + 6x^{-4})(6x^4 + 7)$

13) $f(x) = \left(-7x^4 + 10x^{\frac{2}{5}} + 8\right)(x^2 + 10)$

Critical thinking question:

14) A classmate claims that $(f \cdot g)' = f' \cdot g'$ for any functions f and g. Show an example that proves your classmate wrong.

9) $y = (5x^4 - 3x^2 - 1)(-5x^2 + 3)$

$$\frac{dy}{dx} = (5x^4 - 3x^2 - 1) \cdot -10x + (-5x^2 + 3)(20x^3 - 6x)$$
$$= -150x^5 + 120x^3 - 8x$$

10) $f(x) = \left(-10x^2 - 7\sqrt[5]{x^2} + 9\right)(2x^3 + 4)$

$$f'(x) = \left(-10x^2 - 7x^{\frac{2}{5}} + 9\right) \cdot 6x^2 + (2x^3 + 4)\left(-20x - \frac{14}{5}x^{-\frac{3}{5}}\right)$$
$$= -100x^4 - \frac{238x^{\frac{12}{5}}}{5} + 54x^2 - 80x - \frac{56}{5x^{\frac{3}{5}}}$$

11) $y = (5 + 3x^{-2})(4x^5 + 6x^3 + 10)$

$$\frac{dy}{dx} = (5 + 3x^{-2})(20x^4 + 18x^2) + (4x^5 + 6x^3 + 10) \cdot -6x^{-3}$$
$$= 100x^4 + 126x^2 + 18 - \frac{60}{x^3}$$

12) $y = (-6x^4 + 2 + 6x^{-4})(6x^4 + 7)$

$$\frac{dy}{dx} = (-6x^4 + 2 + 6x^{-4}) \cdot 24x^3 + (6x^4 + 7)(-24x^3 - 24x^{-5})$$
$$= -288x^7 - 120x^3 - \frac{168}{x^5}$$

13) $f(x) = \left(-7x^4 + 10x^{\frac{2}{5}} + 8\right)(x^2 + 10)$

$$f'(x) = \left(-7x^4 + 10x^{\frac{2}{5}} + 8\right) \cdot 2x + (x^2 + 10)\left(-28x^3 + 4x^{-\frac{3}{5}}\right)$$
$$= -42x^5 - 280x^3 + 24x^{\frac{7}{5}} + 16x + \frac{40}{x^{\frac{3}{5}}}$$

Critical thinking question:

14) A classmate claims that $(f \cdot g)' = f' \cdot g'$ for any functions f and g. Show an example that proves your classmate wrong.

Many answers. Ex: $f = 2x$, $g = 4$, $8 \neq 0$

 Name_____

Differentiation - Quotient Rule

Date_____ Period____

Differentiate each function with respect to *x*.

1) $y = \dfrac{2}{2x^4 - 5}$

2) $f(x) = \dfrac{2}{x^5 - 5}$

3) $f(x) = \dfrac{5}{4x^3 + 4}$

4) $y = \dfrac{4x^3 - 3x^2}{4x^5 - 4}$

5) $y = \dfrac{3x^4 + 2}{3x^3 - 2}$

6) $y = \dfrac{4x^5 + 2x^2}{3x^4 + 5}$

7) $y = \dfrac{4x^5 + x^2 + 4}{5x^2 - 2}$

8) $y = \dfrac{3x^4 + 5x^3 - 5}{2x^4 - 4}$

Kuta Software - Infinite Calculus

Name_____

Differentiation - Quotient Rule

Date_____ Period____

Differentiate each function with respect to *x*.

1) $y = \dfrac{2}{2x^4 - 5}$

$\dfrac{dy}{dx} = -\dfrac{2 \cdot 8x^3}{(2x^4 - 5)^2}$

$= -\dfrac{16x^3}{4x^8 - 20x^4 + 25}$

2) $f(x) = \dfrac{2}{x^5 - 5}$

$f'(x) = -\dfrac{2 \cdot 5x^4}{(x^5 - 5)^2}$

$= -\dfrac{10x^4}{x^{10} - 10x^5 + 25}$

3) $f(x) = \dfrac{5}{4x^3 + 4}$

$f'(x) = -\dfrac{5 \cdot 12x^2}{(4x^3 + 4)^2}$

$= -\dfrac{15x^2}{4x^6 + 8x^3 + 4}$

4) $y = \dfrac{4x^3 - 3x^2}{4x^5 - 4}$

$\dfrac{dy}{dx} = \dfrac{(4x^5 - 4)(12x^2 - 6x) - (4x^3 - 3x^2) \cdot 20x^4}{(4x^5 - 4)^2}$

$= \dfrac{-8x^7 + 9x^6 - 12x^2 + 6x}{4x^{10} - 8x^5 + 4}$

5) $y = \dfrac{3x^4 + 2}{3x^3 - 2}$

$\dfrac{dy}{dx} = \dfrac{(3x^3 - 2) \cdot 12x^3 - (3x^4 + 2) \cdot 9x^2}{(3x^3 - 2)^2}$

$= \dfrac{9x^6 - 24x^3 - 18x^2}{9x^6 - 12x^3 + 4}$

6) $y = \dfrac{4x^5 + 2x^2}{3x^4 + 5}$

$\dfrac{dy}{dx} = \dfrac{(3x^4 + 5)(20x^4 + 4x) - (4x^5 + 2x^2) \cdot 12x^3}{(3x^4 + 5)^2}$

$= \dfrac{12x^8 - 12x^5 + 100x^4 + 20x}{9x^8 + 30x^4 + 25}$

7) $y = \dfrac{4x^5 + x^2 + 4}{5x^2 - 2}$

$\dfrac{dy}{dx} = \dfrac{(5x^2 - 2)(20x^4 + 2x) - (4x^5 + x^2 + 4) \cdot 10x}{(5x^2 - 2)^2}$

$= \dfrac{60x^6 - 40x^4 - 44x}{25x^4 - 20x^2 + 4}$

8) $y = \dfrac{3x^4 + 5x^3 - 5}{2x^4 - 4}$

$\dfrac{dy}{dx} = \dfrac{(2x^4 - 4)(12x^3 + 15x^2) - (3x^4 + 5x^3 - 5) \cdot 8x^3}{(2x^4 - 4)^2}$

$= \dfrac{-5x^6 - 4x^3 - 30x^2}{2x^8 - 8x^4 + 8}$

9) $y = \dfrac{x^3 - x^2 - 3}{x^5 + 3}$

10) $y = \dfrac{x^4 + 6}{3 - 4x^{-4}}$

11) $y = \dfrac{4x^4 - 4x^2 + 5}{2x^{\frac{5}{3}} + 3}$

Critical thinking question:

12) A classmate claims that $\left(\dfrac{f}{g}\right)' = \dfrac{f'}{g'}$ for any functions f and g. Show an example that proves your classmate wrong.

9) $y = \dfrac{x^3 - x^2 - 3}{x^5 + 3}$

$$\frac{dy}{dx} = \frac{(x^5 + 3)(3x^2 - 2x) - (x^3 - x^2 - 3) \cdot 5x^4}{(x^5 + 3)^2}$$
$$= \frac{-2x^7 + 3x^6 + 15x^4 + 9x^2 - 6x}{x^{10} + 6x^5 + 9}$$

10) $y = \dfrac{x^4 + 6}{3 - 4x^{-4}}$

$$\frac{dy}{dx} = \frac{(3 - 4x^{-4}) \cdot 4x^3 - (x^4 + 6) \cdot 16x^{-5}}{(3 - 4x^{-4})^2}$$
$$= \frac{12x^{11} - 32x^7 - 96x^3}{9x^8 - 24x^4 + 16}$$

11) $y = \dfrac{4x^4 - 4x^2 + 5}{2x^{\frac{5}{3}} + 3}$

$$\frac{dy}{dx} = \frac{\left(2x^{\frac{5}{3}} + 3\right)(16x^3 - 8x) - (4x^4 - 4x^2 + 5) \cdot \frac{10}{3}x^{\frac{2}{3}}}{\left(2x^{\frac{5}{3}} + 3\right)^2}$$
$$= \frac{56x^{\frac{14}{3}} + 144x^3 - 8x^{\frac{8}{3}} - 72x - 50x^{\frac{2}{3}}}{12x^{\frac{10}{3}} + 36x^{\frac{5}{3}} + 27}$$

Critical thinking question:

12) A classmate claims that $\left(\dfrac{f}{g}\right)' = \dfrac{f'}{g'}$ for any functions f and g. Show an example that proves your classmate wrong.

Many answers. Ex: $f = 4$, $g = 2x$, $-\dfrac{2}{x^2} \neq 0$

TEST - Dr Ahn Math

Find dy/dx.

1. $y = 6x^{1.5} - 4x^{.5}$

2. $y = 8\sqrt{x} + 6x^{3/4}$

3. $y = 6x^{-5} - x^{-1}$

4. $y = \frac{6}{x} - \frac{8}{x^3}$

5. $y = x^{-1/2} - 14x^{-3/2}$

6. $y = \frac{-2}{\sqrt[3]{x}}$

7. $y = 9x^{-1/2} + \frac{2}{x^{3/2}}$

8. $y = \frac{3}{x^6} + \frac{1}{x^5} - \frac{7}{x^2}$

9. $y = -2x^{2.5} + 8x^{.5}$

10. $y = -15x^{3/2} + 2x^{1.9}$

11. $y = -100\sqrt{x} - 11x^{2/3}$

ID: 1

Calculus

Name_____

Assignment

Date_____ Period____

Differentiate each function with respect to x.

1) $y = (-2x^5 - 5)(-4x^2 + 4)$

2) $y = -2x^3(-3x^3 + 5)$

3) $y = (3x^2 - 5) \cdot 2x^4$

7) $y = \dfrac{5}{3x^3 + 4}$

8) $f(x) = \dfrac{x^4 - 5x^3}{3x^3 + 4}$

9) $f(x) = \dfrac{5x^4}{5x^5 + 3}$

10) $f(x) = \dfrac{4x^5 + 2x^4 - 4x^2}{2x^2 + 5}$

Calculus

Name_____

Assignment

Date_____ Period____

Differentiate each function with respect to x.

1) $y = (-2x^5 - 5)(-4x^2 + 4)$

$$\frac{dy}{dx} = (-2x^5 - 5) \cdot -8x + (-4x^2 + 4) \cdot -10x^4$$

2) $y = -2x^3(-3x^3 + 5)$

$$\frac{dy}{dx} = -2x^3 \cdot -9x^2 + (-3x^3 + 5) \cdot -6x^2$$

3) $y = (3x^2 - 5) \cdot 2x^4$

$$\frac{dy}{dx} = (3x^2 - 5) \cdot 8x^3 + 2x^4 \cdot 6x$$

7) $y = \dfrac{5}{3x^3 + 4}$

$$\frac{dy}{dx} = -\frac{5 \cdot 9x^2}{(3x^3 + 4)^2}$$

8) $f(x) = \dfrac{x^4 - 5x^3}{3x^3 + 4}$

$$f'(x) = \frac{(3x^3 + 4)(4x^3 - 15x^2) - (x^4 - 5x^3) \cdot 9x^2}{(3x^3 + 4)^2}$$

9) $f(x) = \dfrac{5x^4}{5x^5 + 3}$

$$f'(x) = \frac{(5x^5 + 3) \cdot 20x^3 - 5x^4 \cdot 25x^4}{(5x^5 + 3)^2}$$

10) $f(x) = \dfrac{4x^5 + 2x^4 - 4x^2}{2x^2 + 5}$

A) $f'(x) = 24x^6 + 8x^5 + 100x^4 + 40x^3 - 40x$

*B) $f'(x) = \dfrac{24x^6 + 8x^5 + 100x^4 + 40x^3 - 40x}{4x^4 + 20x^2 + 25}$

C) $20x^4 + 8x^3 - 12x$

YouTube

< Higher Order Derivatives >

- $y = x^5 + x^4 + x^3$

$$\frac{dy}{dx} = y' =$$

$$\frac{d^2y}{dx^2} = y'' =$$

$$\frac{d^3y}{dx^3} = y''' =$$

$$\frac{d^4y}{dx^4} = y'''' =$$

$$\frac{d^5y}{dx^5} = y^{(5)} =$$

$$\frac{d^6y}{dx^6} = y^{(6)} =$$

< Higher Order Derivatives >

- $y = x^5 + x^4 + x^3$

$$\frac{dy}{dx} = y' = 5x^4 + 4x^3 + 3x^2$$

$$\frac{d^2y}{dx^2} = y'' = 20x^3 + 12x^2 + 6x$$

$$\frac{d^3y}{dx^3} = y''' = 60x^2 + 24x + 6$$

$$\frac{d^4y}{dx^4} = y'''' = 120x + 24$$

$$\frac{d^5y}{dx^5} = y^{(5)} = 120$$

$$\frac{d^6y}{dx^6} = y^{(6)} = 0$$

Kuta Software - Infinite Calculus Name_____

Higher Order Derivatives

Date_____ Period____

For each problem, find the indicated derivative with respect to *x*.

1) $y = -x^2$ Find $\frac{d^2y}{dx^2}$

2) $f(x) = 4x^3$ Find f''

3) $y = -4x$ Find $\frac{d^3y}{dx^3}$

4) $f(x) = 5x^4$ Find f'''

5) $y = -5x^4$ Find $\frac{d^4y}{dx^4}$

6) $y = 3x^5 - 2x$ Find $\frac{d^3y}{dx^3}$

7) $y = -2x^3 - 4x^{-3}$ Find $\frac{d^3y}{dx^3}$

8) $y = -x^2 + 2\sqrt[5]{x^2}$ Find $\frac{d^3y}{dx^3}$

Critical thinking questions. Find the indicated derivatives with respect to *x*.

9) $y = 99x^{99}$ Find $\frac{d^{100}y}{dx^{100}}$

10) $f(x) = x^{99}$ Find $f^{(99)}$

Kuta Software - Infinite Calculus Name_____

Higher Order Derivatives

Date_____ Period____

For each problem, find the indicated derivative with respect to x.

1) $y = -x^2$ Find $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = -2$

2) $f(x) = 4x^3$ Find f''

$f''(x) = 24x$

3) $y = -4x$ Find $\frac{d^3y}{dx^3}$

$\frac{d^3y}{dx^3} = 0$

4) $f(x) = 5x^4$ Find f'''

$f'''(x) = 120x$

5) $y = -5x^4$ Find $\frac{d^4y}{dx^4}$

$\frac{d^4y}{dx^4} = -120$

6) $y = 3x^5 - 2x$ Find $\frac{d^3y}{dx^3}$

$\frac{d^3y}{dx^3} = 180x^2$

7) $y = -2x^3 - 4x^{-3}$ Find $\frac{d^3y}{dx^3}$

$\frac{d^3y}{dx^3} = -12 + \frac{240}{x^6}$

8) $y = -x^2 + 2\sqrt[5]{x^2}$ Find $\frac{d^3y}{dx^3}$

$\frac{d^3y}{dx^3} = \frac{96}{125x^{\frac{13}{5}}}$

Critical thinking questions. Find the indicated derivatives with respect to x.

9) $y = 99x^{99}$ Find $\frac{d^{100}y}{dx^{100}}$

The 99th derivative is a constant, so 100th derivative is 0.

10) $f(x) = x^{99}$ Find $f^{(99)}$

99! (Made easy by factorial notation)